Rotation Between Two Vectors in $\mathbb{R}^3$

This is valid and numerically stable as long as the two vectors a and b are not pointing in opposite directions:

$$
\begin{aligned}
a, b &\in \mathbb{R}^3 \\
\hat{a} &= \frac{a}{\|a\|} \\
\hat{b} &= \frac{b}{\|b\|} \\
\omega &= \hat{a} \times \hat{b} \\
c &= \frac{1}{1 + \hat{a}^T \hat{b}} \\
{}^b R_a &= I_3 + \omega_\times + c\omega_\times^2 \\
&= \begin{pmatrix} 1 - c\left(\omega_2^2 + \omega_3^2\right) & c\omega_1\omega_2 - \omega_3 & c\omega_1\omega_3 + \omega_2 \\ c\omega_1\omega_2 + \omega_3 & 1 - c\left(\omega_1^2 + \omega_3^2\right) & c\omega_2\omega_3 - \omega_1 \\ c\omega_1\omega_3 - \omega_2 & c\omega_2\omega_3 + \omega_1 & 1 - c\left(\omega_1^2 + \omega_2^2\right) \end{pmatrix}
\end{aligned}
$$

When $\hat{a}^T \hat{b} < \epsilon - 1$, the rotation is best parameterized by first flipping around a vector perpendicular to a. Let such a vector be p:

$$
\begin{aligned}
p^T a &= 0 \\
p^T p &= 1
\end{aligned}
$$

The rotation which leaves p fixed and sends $\hat{a}$ to $-\hat{a}$ is then:

$$
{}^{-a} R_a = 2pp^T - I_3
$$

Composing this with ${}^b R_{-a}$ yields the desired rotation, avoiding singularities:

$$
\begin{aligned}
{}^b R_a &= {}^b R_{-a} \cdot {}^{-a} R_a \\
&= \left(I_3 - \omega_\times + \frac{1}{1 - \hat{a}^T \hat{b}} \cdot \omega_\times^2 \right) \cdot \left(2pp^T - I_3\right)
\end{aligned}
$$